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Derivation of the effective potential for quantum electrodynamics

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Abstract. The functional formalism for the effective potential is briefly reviewed for the case of a scalar field theory, using the method of steepest descents. This formalism is then applied to quantum electrodynamics and an integral expression is derived for the effective potential, in the one-loop approximation. This expression is used to verify the absence of spontaneous symmetry breakdown for quantum electrodynamics in one space and one time dimension.

1. Introduction

Spontaneous symmetry breakdown for a given field theory may be investigated by calculating the effective potential from the Lagrangian density (Coleman and Weinberg 1973, Coleman 1975). The behaviour of the derivative of the effective potential with respect to the 'classical field' determines whether the symmetry of the Lagrangian density is spontaneously broken or not. In this paper the effective potential for quantum electrodynamics is considered, i.e. an explicit expression is derived for the effective potential in the one-loop approximation; this expression is then used to show that, to this order at least, there is no spontaneous symmetry breakdown for the case of quantum electrodynamics in one space and one time dimension, which is the expected result.

This introduction reviews briefly the functional formalism developed for the effective potential, in the context of a scalar field theory (Coleman and Weinberg 1973, Coleman 1975, Iliopoulos *et al* 1975); in the remainder of the paper this formalism is applied to the more interesting case of quantum electrodynamics.

The effective potential is defined in terms of the effective action; consider the Lagrangian density for a scalar field

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu\phi)^2-\tfrac{1}{2}\mu^2\phi^2-(\lambda/4!)\phi^4-J\phi \tag{1.1}$$

where $J(x)$ is some external source, a c-number function of space and time.

The connected generating functional $W[J]$ is defined by

$$e^{iW[J]}\equiv\langle 0^+|0^-\rangle_J \tag{1.2}$$

i.e. it is the vacuum-to-vacuum amplitude in the presence of the source $J(x)$, $W[J]$ generates connected Feynman diagrams.

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The effective action $\Gamma[\phi_c]$ is defined by a functional Legendre transformation:

$$\Gamma[\phi_c] \equiv W[J] - \int d^4x\, J(x)\phi_c(x) \tag{1.3}$$

where $\phi_c(x)$—the 'classical field'—is given by

$$\phi_c(x) \equiv \delta W[J]/\delta J(x). \tag{1.4}$$

Equation (1.3) implies that

$$\delta\Gamma/\delta\phi_c(x) = -J(x). \tag{1.5}$$

It is well known that $\Gamma[\phi_c]$ generates one-particle irreducible Feynman diagrams (Coleman and Weinberg 1973, Coleman 1975, Abers and Lee 1973, chaps 11, 12 and 16).

The effective action may also be expanded in powers of the derivative of ϕ_c around the point ϕ_c = constant:

$$\Gamma[\phi_c] = \int d^4x[-V(\phi_c) + \tfrac{1}{2}(\partial_\mu\phi_c)^2 Z(\phi_c) + \dots] \tag{1.6}$$

where the function $V(\phi_c)$ is the effective potential.

The Lagrangian density (1.1) is invariant under the transformation $\phi \to -\phi$ provided that J is set equal to zero; spontaneous symmetry breaking occurs if ϕ develops a non-zero vacuum expectation value, i.e. ϕ_c as defined in equation (1.4) is non-zero; since the source J is zero this corresponds to

$$\delta\Gamma/\delta\phi_c(x) = 0, \qquad \phi_c \neq 0. \tag{1.7}$$

The vacuum expectation value is usually taken to be translation invariant so that equations (1.6) and (1.7) immediately lead to

$$dV/d\phi_c = 0, \qquad \phi_c \neq 0. \tag{1.8}$$

Thus, spontaneous symmetry breakdown can be investigated by examining the minima of the effective potential $V(\phi_c)$.

The evaluation of $V(\phi_c)$ involves an infinite summation of graphs; however, a suitable approximation scheme has been developed, known as the $\hbar$ expansion, which involves the method of steepest descents (Iliopoulos et al 1975). The vacuum-to-vacuum amplitude is written as a Feynman path integral:

$$\langle 0^+|0^-\rangle_J = e^{iW[J]} = \frac{\int [d\phi] \exp[i\hbar^{-1}\int d^4x(\mathscr{L}_0(\phi, \partial_\mu\phi) + J\phi)]}{\int [d\phi] \exp[i\hbar^{-1}\int d^4x \mathscr{L}_0(\phi, \partial_\mu\phi)]} \tag{1.9}$$

with

$$\mathscr{L}_0(\phi, \partial_\mu\phi) = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - (\lambda/4!)\phi^4. \tag{1.10}$$

The exponent in the numerator of the right-hand side of equation (1.9) is expanded around the point $\phi_0[J]$ at which it is stationary. This leads to an expansion in powers of $\hbar$ for $W[J]$ and hence, through equations (1.3) and (1.6), to a corresponding series for $V(\phi_c)$.

The effective potential can be given a physical interpretation. It is the expectation value of the Hamiltonian density in a state $|a\rangle$ for which $\langle a|\mathscr{H}|a\rangle$ is stationary and which is constrained to satisfy $\langle a|a\rangle = 1$ and $\langle a|\phi|a\rangle = \phi_c$ where ϕ is some (relevant) field; normally $|a\rangle$ is the vacuum.

In the following sections, the formalism described above is applied to a theory with more physical content, namely quantum electrodynamics.

2. The effective potential for quantum electrodynamics

2.1. The path integral

The vacuum-to-vacuum amplitude for quantum electrodynamics is given by the path integral expression:

$$\langle 0^+|0^-\rangle_{\eta,\bar{\eta},J} = \frac{\int [\mathrm{d}\psi][\mathrm{d}\bar{\psi}][\mathrm{d}A_\mu]\exp(\mathrm{i}\hbar^{-1}\int \mathscr{L}\,\mathrm{d}^4x)}{\int [\mathrm{d}\psi][\mathrm{d}\bar{\psi}][\mathrm{d}A_\mu]\exp(\mathrm{i}\hbar^{-1}\int \mathscr{L}_{\text{free}}\,\mathrm{d}^4x)} \tag{2.1}$$

where

$$\mathscr{L}_{\text{free}} = \bar{\psi}(\mathrm{i}\not{\partial}-\mu)\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - (\partial\,.\,A)^2/2\xi \tag{2.2}$$

with $(\partial\,.\,A)^2/2\xi$ a gauge-fixing term and

$$\mathscr{L} = \mathscr{L}_{\text{free}} + \mathscr{L}' \tag{2.3}$$

with

$$\mathscr{L}' = -e\bar{\psi}\not{A}\psi + \bar{\eta}\psi + \bar{\psi}\eta + J_\mu A^\mu \tag{2.4}$$

where η, $\bar{\eta}$, J_μ are source terms coupling to the spinor fields $\bar{\psi}$, ψ and the photon field A_μ.

The method of steepest descents consists in expanding the exponent in the numerator of the path integral expression around the points $\psi_0[\bar{\eta}]$, $\bar{\psi}_0[\eta]$, $A^0_\mu[J]$ at which it is stationary. These points must satisfy the equations of motion following from the Lagrangian density (2.3); this leads to the three coupled equations:

$$\mathrm{i}(\not{\partial}-\mu)\psi_0 + \eta = e\not{A}_0\psi_0 \tag{2.5}$$

$$\bar{\psi}_0(-\mathrm{i}\overleftarrow{\not{\partial}}-\mu) + \bar{\eta} = e\bar{\psi}_0\not{A}_0 \tag{2.6}$$

$$\partial_\beta(F^0_{\beta\alpha}) + J_\alpha + \partial_\alpha(\partial\,.\,A_0)/\xi = e\bar{\psi}_0\gamma_\alpha\psi_0. \tag{2.7}$$

The expansion is carried out by the substitution

$$\psi = \psi_0 + \tilde{\psi}, \qquad \bar{\psi} = \bar{\psi}_0 + \tilde{\bar{\psi}}, \qquad A_\mu = A^0_\mu + \tilde{A}_\mu. \tag{2.8}$$

The resulting expression for $\mathscr{L}$ is

$$\begin{aligned}\mathscr{L} = \mathscr{L}_0 &+ \tilde{\bar{\psi}}(\mathrm{i}\not{\partial}-\mu)\psi - e\tilde{\bar{\psi}}\tilde{\not{A}}\tilde{\psi} - \tfrac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - (\partial\,.\,\tilde{A})^2/2\xi \\ &- e\bar{\psi}_0\tilde{\not{A}}\tilde{\psi} - e\tilde{\bar{\psi}}\not{A}_0\tilde{\psi} - e\tilde{\bar{\psi}}\tilde{\not{A}}\psi_0\end{aligned} \tag{2.9}$$

where

$$\mathscr{L}_0 = \bar{\psi}_0(\mathrm{i}\not{\partial}-\mu)\psi_0 - e\bar{\psi}_0\not{A}\psi_0 - \tfrac{1}{4}F^0_{\mu\nu}F_0^{\mu\nu} - (\partial\,.\,A_0)^2/2\xi + \bar{\eta}\psi_0 + \bar{\psi}_0\eta + J_\mu A_0^\mu. \tag{2.10}$$

Equation (2.9) is derived by using the above equations of motion to eliminate terms and by dropping two divergence terms.

Some of the terms in equation (2.9) can be grouped into the form $\bar{\phi}K\phi$:

$$\begin{aligned}\mathscr{L} = \mathscr{L}_0 &+ \bar{\phi}K\phi - \tfrac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - e^2(\bar{\psi}_0\tilde{\not{A}})K^{-1}(\tilde{\not{A}}\psi_0) - (\partial\,.\,\tilde{A})^2/2\xi \\ &- e(\bar{\phi}+\bar{\chi})\not{A}_0(\phi+\chi) - e(\bar{\phi}+\bar{\chi})\tilde{\not{A}}(\phi+\chi)\end{aligned} \tag{2.11}$$

where $\tilde{\psi}$ has been written

$$\tilde{\psi} = \phi + \chi \tag{2.12}$$

and K and χ are connected by the choice

$$K\chi = e\tilde{\not{A}}\psi_0. \tag{2.13}$$

The freedom of gauge may be used to put $\not{A}_0 = 0$ and the last term in equation (2.11) will not be considered since when $\mathscr{L}$ is rescaled with $\psi \to \hbar^{1/2}\psi$ etc, this term is of order $\hbar^3$.

Equation (2.11) may be made more symmetric by using the following three equations (Abers and Lee 1973, chap. 14):

$$-\tfrac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = \int \mathrm{d}^4y\, \tfrac{1}{2}\tilde{A}_\mu(x)K^{\mu\nu}(x, y)\tilde{A}_\nu(y) \tag{2.14}$$

where

$$K^{\mu\nu}(x, y) = -(\partial^2 g^{\mu\nu} - \partial^\mu\partial^\nu)\,\delta^4(x-y)$$
$$-\frac{1}{2\xi}(\partial\,.\,\tilde{A})^2 = -\frac{1}{2\xi}\int \mathrm{d}^4y\, \tilde{A}_\mu(x)\,\partial^\mu\partial^\nu A_\nu(y)\,\delta^4(x-y) \tag{2.15}$$

and

$$e^2(\bar{\psi}_0\tilde{\not{A}})K^{-1}(\tilde{\not{A}}\psi_0) = e^2\int \mathrm{d}^4y\, \tilde{A}_\mu(x)(\bar{\psi}_0(x)\gamma^\mu K^{-1}\,.\,\gamma^\nu\psi_0(y))\hat{A}_\nu(y)\,\delta^4(x-y). \tag{2.16}$$

The path integral expression (2.1) now is

$$\exp\left(\frac{\mathrm{i}}{\hbar}W[\eta, \bar{\eta}, J]\right) = \frac{\int[\mathrm{d}\psi][\mathrm{d}\bar{\psi}][\mathrm{d}A_\mu]\exp(\mathrm{i}\hbar^{-1}\int\mathscr{L}\,\mathrm{d}^4x)}{\int[\mathrm{d}\psi][\mathrm{d}\bar{\psi}][\mathrm{d}A_\mu]\exp(\mathrm{i}\hbar^{-1}\int\mathscr{L}_{\mathrm{free}}\,\mathrm{d}^4x)} \tag{2.17}$$

where

$$\mathscr{L} = \mathscr{L}_0 + \bar{\phi}K\phi - \int \mathrm{d}^4y\, \tilde{A}_\mu(x)M^{\mu\nu}(x, y)\hat{A}_\nu(y) \tag{2.18}$$

with

$$\mathscr{L}_0 = \bar{\psi}_0(\mathrm{i}\not{\partial} - \mu)\psi_0 + \bar{\eta}\psi_0 + \bar{\psi}_0\eta \tag{2.19}$$

and

$$M^{\mu\nu}(x, y) = \{-\tfrac{1}{2}[\partial^2 g^{\mu\nu} - \partial^\mu\partial^\nu(1+\xi^{-1})] + e^2\bar{\psi}_0(x)\gamma^\mu K^{-1}\gamma^\nu\psi_0(y)\}\,\delta^4(x-y). \tag{2.20}$$

2.2. The $\hbar$ expansions

2.2.1. The $\hbar$ series for the connected generating functional is

$$W[\eta, \bar{\eta}, J] = W_0[\eta, \bar{\eta}, J] + \hbar W_1[\psi_0, \bar{\psi}_0] + \hbar^2 W_2[\psi_0, \bar{\psi}_0] + \ldots \tag{2.21}$$

where

$$W_0[\eta, \bar{\eta}, J] = \int \mathrm{d}^4x[\bar{\psi}_0(\mathrm{i}\not{\partial} - \mu)\psi_0 + \bar{\eta}\psi_0 + \bar{\psi}_0\eta]. \tag{2.22}$$

The path integral expression (2.17) becomes

$$\exp[\mathrm{i}\hbar^{-1}(W-W_0)]$$
$$=\frac{\int[\mathrm{d}\phi][\mathrm{d}\bar\phi][\mathrm{d}A_\mu]\exp[\mathrm{i}\hbar^{-1}\int\mathrm{d}^4x(\bar\phi(x)K\phi(x)-\int\mathrm{d}^4y\,A_\mu(x)M^{\mu\nu}(x,y)A_\nu(y))]}{\int[\mathrm{d}\psi][\mathrm{d}\bar\psi][\mathrm{d}A_\mu]\exp\{\mathrm{i}\hbar^{-1}\int\mathrm{d}^4x[\bar\psi(\mathrm{i}\not\partial-\mu)\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-(\partial\,.\,A)^2/2\xi]\}} \qquad (2.23)$$

where the tilde on the A_μ have been dropped since $A^0_\mu=0$.

The new integration variables in equation (2.23), obtained from the translation (2.12), do not change the path integral, since functional integration is translation invariant. The quadratic forms in the exponents of the numerator and denominator give $W_1[\psi_0,\bar\psi_0]$ when the fields are rescaled by a factor $\hbar^{1/2}$. The A_μ-field integration is similar to a scalar field integration, if θ is a scalar field, then (Coleman 1975, chap. 4)

$$\int[\mathrm{d}\theta]\exp(\mathrm{i}\hbar^{-1}\theta M\theta)=(\det M)^{-1/2}. \qquad (2.24)$$

The ϕ and $\bar\phi$ integrations require more care since ψ and $\bar\psi$ and equivalently ϕ and $\bar\phi$ are anticommuting objects. For such anticommuting objects, the functional integral turns out to be (Berezin 1966, Coleman 1975, chap. 4)

$$\int[\mathrm{d}\psi][\mathrm{d}\bar\psi]\exp(\mathrm{i}\hbar^{-1}\bar\psi K\psi)=\det K. \qquad (2.25)$$

All this leads to

$$\mathrm{i}\,W[\psi_0,\bar\psi_0]=\ln\left(\frac{\det K(\det M)^{-1/2}}{\det K(\det M_0)^{-1/2}}\right) \qquad (2.26)$$

where

$$M^{\mu\nu}(x,y)=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\left\{-k^2\left[g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\left(1+\frac{1}{\xi}\right)\right]+e^2\bar\psi_0\gamma^\mu\frac{1}{k-\mu}\gamma^\nu\psi_0\right\}\mathrm{e}^{\mathrm{i}k(x-y)} \qquad (2.27)$$

and

$$M_0^{\mu\nu}(x,y)=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\left\{-k^2\left[g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\left(1+\frac{1}{\xi}\right)\right]\right\}\mathrm{e}^{\mathrm{i}k(x-y)}. \qquad (2.28)$$

Then,

$$W_1=-\mathrm{i}\ln\det(MM_0^{-1})^{-1/2}=\tfrac{1}{2}\mathrm{i}\,\mathrm{Tr}\ln(MM_0^{-1}) \qquad (2.29)$$

where M_0^{-1} must be of the form

$$(M_0^{-1})_{\nu\sigma}=-(ag_{\nu\sigma}-bk_\nu k_\sigma)/k^2. \qquad (2.30)$$

The requirement

$$M_0M_0^{-1}=1 \qquad (2.31)$$

leads to

$$a=1,\qquad b=\xi(1+\xi^{-1})/k^2. \qquad (2.32)$$

2.2.2. The effective action is given by

$$\Gamma[\psi_c,\bar\psi_c,A^c_\mu]\equiv W[\eta,\bar\eta,J]-\int\mathrm{d}^4x(\bar\eta(x)\psi_c(x)+\bar\psi_c(x)\eta(x)+J_\mu(x)A^\mu(x)) \qquad (2.33)$$

where the 'classical fields' $\psi_c(x)$, $\bar\psi_c(x)$ and $A^c_\mu(x)$ are defined by

$$\psi_c(x)=\delta W/\delta\bar\eta(x), \qquad \bar\psi_c(x)=\delta W/\delta\eta(x), \qquad A^c_\mu(x)=\delta W/\delta J_\mu(x). \tag{2.34}$$

From equations (2.21) and (2.22),

$$\psi_c(x)=\frac{\delta W_0}{\delta\bar\eta(x)}+\mathrm{O}(\hbar)=\psi_0(x)+\mathrm{O}(\hbar). \tag{2.35}$$

Similarly,

$$\bar\psi_c(x)=\bar\psi_0(x)+\mathrm{O}(\hbar), \qquad A^c_\mu(x)=\mathrm{O}(\hbar) \tag{2.36}$$

since A^0_μ was set equal to zero.

The $\hbar$ expansion for Γ is

$$\Gamma(\psi_c,\bar\psi_c,A^c_\mu]=\Gamma_0+\hbar\Gamma_1+\hbar^2\Gamma_2+\dots. \tag{2.37}$$

The Γ_0 term is given by equations (2.22) and (2.23) with the use of equations (2.35) and (2.36):

$$\Gamma_0=\int \mathrm{d}^4x\,\bar\psi_0(\mathrm{i}\not\partial-\mu)\psi_0. \tag{2.38}$$

Equations (2.35) and (2.36) also give

$$\psi_c=\psi_0+\hbar\tilde\psi_c \qquad \bar\psi_c=\bar\psi_0+\hbar\tilde{\bar\psi}_c \qquad A^c_\mu=\hbar\tilde A^c_\mu. \tag{2.39}$$

It then follows that

$$W[\eta,\bar\eta,J]-W_0[\eta,\bar\eta,J]=\hbar W_1[\psi_c-\hbar\tilde\psi_c,\bar\psi_c-\hbar\tilde{\bar\psi}_c]+\dots \tag{2.40}$$

and so

$$\Gamma_1=W_1=\tfrac{1}{2}\mathrm{i}\,\mathrm{Tr}\ln(MM_0^{-1}). \tag{2.41}$$

2.2.3. The effective potential is defined as in equation (1.6)

$$\Gamma[\psi_c,\bar\psi_c,A^c_\mu]=\int \mathrm{d}^4x(-V(\psi_c,\bar\psi_c,A^c_\mu)+\dots) \tag{2.42}$$

and the $\hbar$ expansion for V is

$$V=V_0+\hbar V_1+\hbar^2V_2+\dots. \tag{2.43}$$

Each term in the V series is obtained from the Γ series by making ψ_c, $\bar\psi_c$ and A^c_μ all constant and dropping the $\int \mathrm{d}^4x$ factor. The first term V_0 is then, from equation (2.38),

$$V_0=\mu\bar\psi_0\psi_0. \tag{2.44}$$

The next term V_1 is obtained from Γ_1 and equation (2.41) leads to

$$V_1=-\frac{\mathrm{i}}{2}\mathrm{Tr}\int \mathrm{d}^4k\,\ln\bigg(\bigg\{-k^2\bigg[g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\bigg(1+\frac{1}{\xi}\bigg)\bigg]+e^2\bar\psi_0\gamma^\mu\frac{(\not k+\mu)}{k^2-\mu^2}\gamma^\nu\psi_0\bigg\}$$
$$\times\bigg[-\frac{g_{\nu\sigma}}{k^2}+\frac{k_\nu k_\sigma\xi}{k^4}\bigg(1+\frac{1}{\xi}\bigg)\bigg]\bigg). \tag{2.45}$$

The next step is to take the trace, but the four-dimensional integral in equation (2.45) is not easy to evaluate. The calculation of V_1 is best illustrated by considering the simpler case of quantum electrodynamics in one space and one time dimension.

3. Quantum electrodynamics in one space and one time dimension

In one space and one time dimension the integrand of equation (2.45) is unchanged in form. The trace is obtained as follows:

$$\bar{\psi}_0\gamma^\mu(K\!\!\!/+\mu)\gamma^\nu\psi_0 = -(\psi_0)_\beta(\bar{\psi}_0)_\alpha[\gamma^\mu(K\!\!\!/+\mu)\gamma^\nu]_{\alpha\beta} \tag{3.1}$$

since, for spinors,

$$(\bar{\psi}_0)_\alpha(\psi_0)_\beta = -(\psi_0)_\beta(\bar{\psi}_0)_\alpha. \tag{3.2}$$

An expression such as $\psi\bar{\psi}$ may be expressed in terms of the set of four linearly independent 2×2 matrices formed from the standard γ matrices derived from the Dirac equation for one space and one time dimension,

$$(p\!\!\!/-\mu)\psi = 0 \tag{3.3}$$

where

$$p\!\!\!/ = \gamma^\mu p_\mu = \gamma^\mu \mathrm{i}(\partial/\partial x^\mu), \qquad \hbar = c = 1. \tag{3.4}$$

The γ satisfy

$$\{\gamma^\mu\gamma^\nu\} = 2g^{\mu\nu}1, \quad \mu, \nu = 0, 1 \qquad g^{\mu\nu} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{3.5}$$

These four matrices are

$$1, \gamma^\mu, \gamma^0, \gamma^1. \tag{3.6}$$

In terms of these,

$$\psi\bar{\psi} = a\,1 + b_\mu\gamma^\mu + c\gamma^0\gamma^1. \tag{3.7}$$

A particular representation is

$$\gamma^0 = \sigma^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad \gamma^1 = \mathrm{i}\sigma^3 = \begin{pmatrix}\mathrm{i} & 0\\ 0 & -\mathrm{i}\end{pmatrix} \tag{3.8}$$

where σ^i are the Pauli matrices.

Substitution of equation (3.7) into equation (3.1) leads to

$$\bar{\psi}_0\gamma^\mu(K\!\!\!/+\mu)\gamma^\nu\psi_0$$
$$= -a\,\mathrm{Tr}[\gamma^\mu(K\!\!\!/+\mu)\gamma^\nu] - b_\rho\,\mathrm{Tr}[\gamma^\rho\gamma^\mu(K\!\!\!/+\mu\gamma^\nu] - c\,\mathrm{Tr}[\gamma^0\gamma^1\gamma^\mu(K\!\!\!/+\mu)\gamma^\nu]. \tag{3.9}$$

The traces may be calculated with the use of standard trace theorems (Bjorken and Drell 1964); restricted to one space and one time dimension the result is

$$\bar{\psi}_0\gamma^\mu(K\!\!\!/+\mu)\gamma^\nu\psi_0 = -2\mu a g^{\mu\nu} - 2(b^\mu k^\nu + b^\nu k^\mu - b\,.\,k g^{\mu\nu}) - 2\mu c\epsilon^{\mu\nu} \tag{3.10}$$

where

$$b^\mu = g^{\mu\sigma}b_\sigma \qquad b\,.\,k = b^0k^0 - b^1k^1 \qquad \epsilon^{\mu\nu} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}. \tag{3.11}$$

Equation (2.45) for V_1 now becomes, using equation (3.10) and with the choice $\xi = -1$:

$$V_1 = -\frac{i}{2}\mathrm{Tr}\int \frac{d^2k}{(2\pi)^2} \ln\left[\left(1+\frac{2e^2(\mu a - b\,.\,k)}{k^2(k^2-\mu^2)}\right)\delta_{\mu\sigma} + \frac{2e^2}{k^2(k^2-\mu^2)}(b^\mu k_\sigma + b_\sigma k^\mu + \mu c\epsilon^{\mu\nu} g_{\nu\sigma})\right]. \tag{3.12}$$

The second term in the integrand can be diagonalised by finding its eigenvalues, which are

$$\lambda = b\,.\,k \pm (b^2k^2 + \mu^2c^2)^{1/2} \tag{3.13}$$

with

$$b^2 = (b^0)^2 - (b^1)^2 \qquad k^2 = (k^0)^2 - (k^1)^2. \tag{3.14}$$

The trace in equation (3.12) may now be taken,

$$V_1 = -\frac{i}{2}\int \frac{d^2k}{(2\pi)^2} \ln\left(1 + \frac{2e^2}{k^2(k^2-\mu^2)}[\mu a \pm (b^2k^2+\mu^2c^2)^{1/2}]\right). \tag{3.15}$$

A compact notation is used here, V_1 is the sum of two terms, the first corresponding to the sum of μa and the square-root term, the second to the difference.

The integrals can be evaluated by changing to a Euclidean metric; k^0 is continued analytically to ik^0; this corresponds to a rotation in the complex k^0 plane. This rotation is allowed since it does not cross the singularities of the integrand in equation (3.15).

To keep the square-root term unchanged, define

$$d^0 = ib^0 \qquad d^1 = b^1. \tag{3.16}$$

Then

$$V_1 = -\frac{1}{2}\int \frac{dk^0 dk^1}{(2\pi)^2} \ln\left(1 + \frac{2e^2}{k^2(k^2+\mu^2)}[\mu a \pm (d^2k^2+\mu^2c^2)^{1/2}]\right) \tag{3.17}$$

where now

$$k^2 = (k^0)^2 + (k^1)^2 \qquad d^2 = (d^0)^2 + (d^1)^2. \tag{3.18}$$

A change in polar coordinates can be made and the angular integration performed,

$$V_1 = -\frac{1}{4\pi}\lim_{\Lambda\to\infty}\int_0^\Lambda dk\, k \ln\left(1 + \frac{2e^2}{k^2(k^2+\mu^2)}[\mu a \pm (d^2k^2+\mu^2c^2)^{1/2}]\right) \tag{3.19}$$

where the limits on the momentum have been introduced.

To make the integration simpler, only the massless case will be considered here. Note that for mass μ equal to zero it follows from equation (2.44) that

$$V_0 = 0. \tag{3.20}$$

Equation (3.19) reduces to

$$V_1 = -\frac{1}{4\pi}\lim_{\Lambda\to\infty}\left(\int_0^\Lambda dk\, k \ln[(k^3+2e^2d)(k^3-2e^2d)] - 2\int_0^\Lambda dk\, k \ln k^3\right). \tag{3.21}$$

The integrations are straightforward, all Λ-dependant terms cancel out, with the result

$$V_1 = \frac{1}{1b\sqrt{3}}(4e^4 d^2)^{1/3} \tag{3.22}$$

with d^2 defined by equations (3.16) and (3.18).

The effective potential may now be plotted using equations (3.20) and (3.22); the plot is shown in figure 1. The vertical scale is in units of $2e$.

The minimum occurs when d is zero; there is no spontaneous symmetry breaking, which is, of course, the expected result.

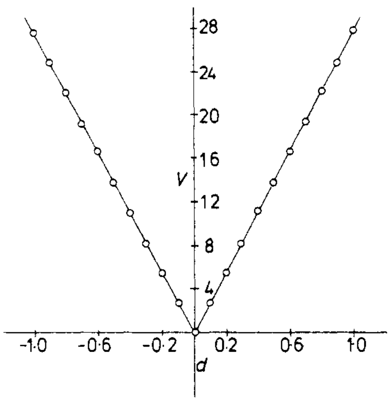

Figure 1. The effective potential in one space and one time dimension.

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